3.1

Exponential and Logistic Functions

The function $f(x) = x^2$ and $g(x) = 2^x$ each involve a base raised to a power, but the roles are reversed.

For $f(x) = x^2$, the base is the variable x, and the exponent is the constant 2.

For $g(x) = 2^x$, the base is the constant 2, and the exponent is the variable x.

The function *f* is a familiar monomial and power function. The function g is an **exponential** function.

Exponential Functions:

Let ***a*** and ***b*** be real number constants. An **exponential function** in ***x*** is a function that can be written in the form:

$$\boldsymbol{f(x) = a \bullet b^x},$$

where ***a*** is nonzero, ***b*** is positive and $b \neq 1$. The constant ***a*** is the ***initial value*** of ***f*** (the value at $x = 0$), and **b** is the **base**.

Objectives:

Identify exponential functions (Example 1 pg. 277)

Compute exponential function values for rational number inputs (Example 2).

Find an exponential function from its table of values (Example 3 pg. 278).

Transforming Exponential Functions (Example 4 pg. 280).

Exponential Growth and Decay:

For any exponential function $f(x) = \boldsymbol{a \bullet b^x}$ and any real number x,

$$f(x+1) = \boldsymbol{b} \bullet f(x).$$

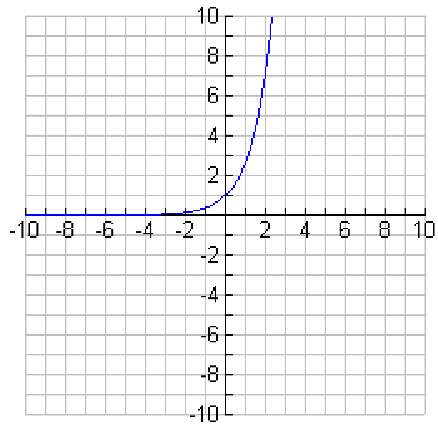
If $a > 0$ and $b > 1$, the function *f* is increasing and is an **exponential growth function**. The base b is its **growth factor.**

If $a > 0$ and $b < 1$, the function f is decreasing and is an **exponential decay function**. The base b is its **decay factor.**

The Natural Base *e*

$f(x) = e^x$

Domain: All reals



Range: $(0,\infty)$

Continuous

Increasing for all x.

No symmetry

Bounded below, but not above

No local extrema

H. Asym. y=0 No V. Asymptote

End behavior: $\lim_{x\to-\infty} e^x = 0$ *and* $\lim_{x\to\infty} e^x = \infty$

<u>The Natural Base *e:*</u>

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

e being named after Leonhard Euler (1707-1783). Because *f(x) = e* has special calculus properties that simplify many calculations, *e* is the *natural base* of exponential functions for calculus purposes, and $f(x) = e^x$ is considered the *natural exponential function*.

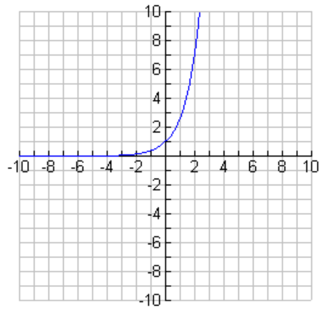
<u>Theorem: Exponential Functions and the Base e:</u>

Any exponential function $f(x) = \boldsymbol{a \bullet b^x}$ can be rewritten as

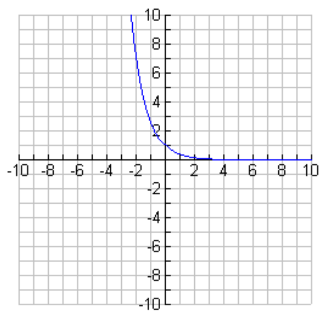
$$f(x) = \boldsymbol{a \bullet e^{kx}} \text{ ,}$$

for an appropriately chosen real number constant *k*.

If *a* > 0 and *k* > 0, *f(x)* = $\boldsymbol{a \bullet e^{kx}}$ is an exponential growth function.



If *a* > 0 and *k* < 0, *f(x)* = $\boldsymbol{a \bullet e^{kx}}$ is an exponential decay function.



<u>Definition: Logistic Growth Functions</u>:

Let *a, b, c*, and *k* be positive constants, with *b* < 1. A **logistic growth function** in *x* is a function that can be written in the form

$$f(x) = \frac{c}{1 + a \bullet b^x} \quad or \quad f(x) = \frac{c}{1 + a \bullet e^{-kx}}$$

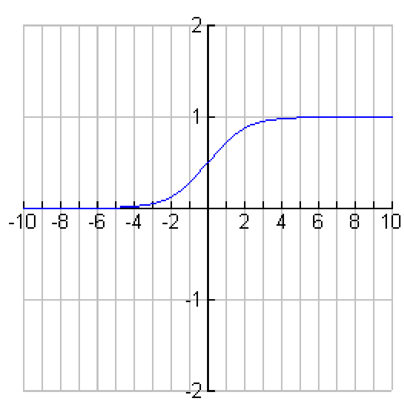
where the constant ***c* is the limit to growth**.

<u>The Logistic Function:</u>

$$f(x) = \frac{1}{1 + e^{-x}}$$

Domain: All reals

Range: (0,1)



Continuous

Increasing for all x.

Symmetric about (0, ½), but not even or odd

Bounded below, and above

No local extrema

H. Asym. y = 0 and y = 1

No V. Asymptote

End behavior: $\lim_{x \to -\infty} f(x) = 0 \quad and \quad \lim_{x \to \infty} f(x) = 1$

See Example 8 pg. 285.